भारतीय प्रौद्योगिकी संस्थान मुंबई

Indian Institute of Technology Bombay

Expand and Conquer

IE 619 Ruleset Presentation

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ज्ञानम् परमम् ध्येयम्

Knowledge is the supreme goal

The game setup

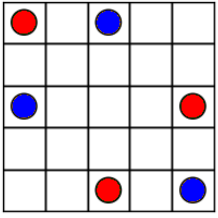

Figure: Example starting position

Start on $m * n$ grid, blue and red pieces across the board

The rules of the game

Definition

(Connected Group) The set of orthogonally adjacent pieces of same color

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(Expanding a Group) Filling in all the orthogonally adjacent squares to your chosen connected group.

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(Expanding a Group) Filling in all the orthogonally adjacent squares to your chosen connected group.

- (Move) Choose a group and expand your group

Example Move

Figure: blue choosing to expand the top group

How does it end?

Figure: blue choosing to expand the top group

The game ends, since red no longer has any move to play, and blue wins

Let's play one then

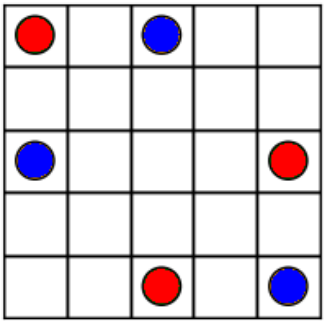

Figure: Lets play this one

Here (Google Collab link)

Integers

See the given position-

Figure: What do you think the game value is

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Hint: Think inductively

Integers

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Figure: What do you think the game value is

Hint: Think inductively
Spoiler: G=5, why?

Dyadics

Figure: Guess the game value

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Okay guess this one first-

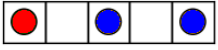

Figure: Hint: Try to think options of both players

Dyadics

Figure: Guess the game value

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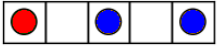

Figure: Hint: Try to think options of both players

The below one is $1/2$, why?

Dyadics

Figure: Guess the game value

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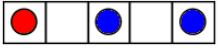


Figure: Hint: Try to think options of both players

The below one is $1/2$, why?
The above one is $1/8$, why?

Disjunctive Sum

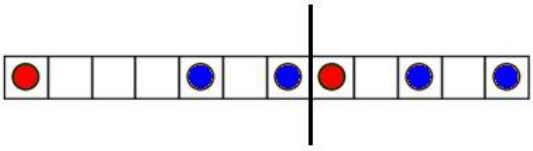


Figure: Hint: Look at the slide title

Disjunctive Sum

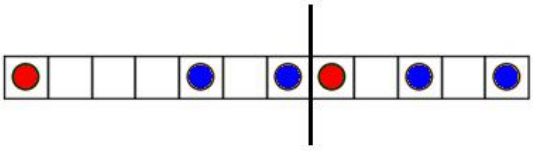


Figure: Hint: Look at the slide title

G=1/8+1/2=5/8

All-Small Version

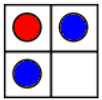
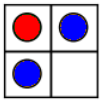


Figure: Can red move?

All-Small Version



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Same rules but one more power

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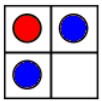

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You can put a single piece at a square not accessible by any of your groups using Rule-1

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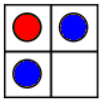

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Where else can red move now?

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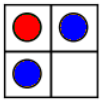

Figure: Can red move?

Same rules but one more power
You can put a single piece at a square not accessible by any of your groups using Rule-1
Where else can red move now? Yes! in the bottom right corner

Example All-Small game

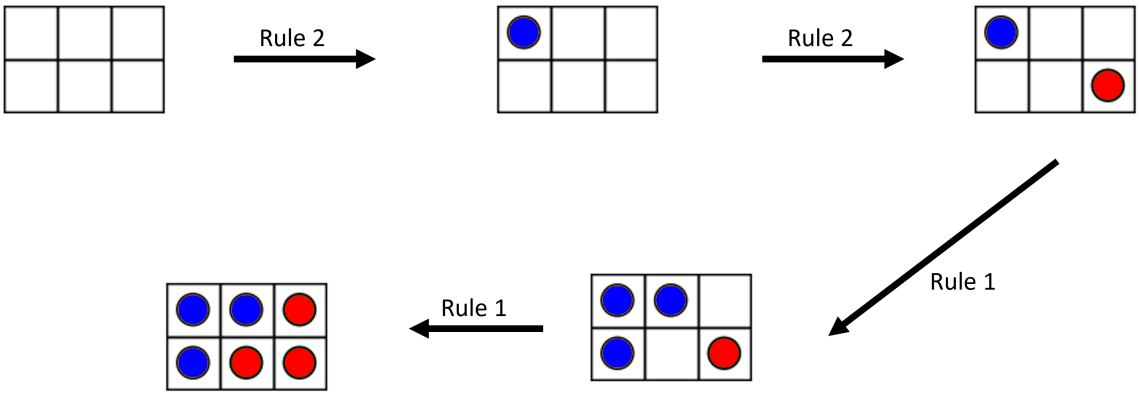

Figure: An example allsmall game, Rule 1 means the normal expand rule, rule 2 means the all-small rule. Notice red wins

Infinitesimals

Figure: Guess the game value (Assume All-Small), Hint again is the slide title

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Figure: Guess the game value (Assume All-Small), Hint again is the slide title

Answer: $\{0 \mid *\} = \uparrow$

A pattern

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g_2	$\downarrow$	-1
g_3	$\uparrow\uparrow$	2
g_4	0	0
g_5	$\uparrow\uparrow\uparrow$	3
g_6	$\uparrow\uparrow\uparrow *$	3
g_7	$\uparrow 4$	4

Table: Games of form g_n

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Let g_n = 'X*O*O$\cdots$ O*' where number of 'O's is n (allsmall) , Then

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Conjecture

$g_{2n-1} = \uparrow n$ where g_k is defined as above.

One more Conjecture

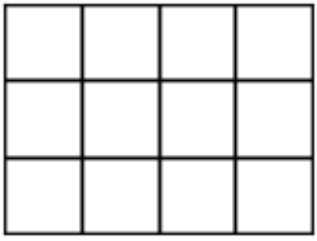

Figure: Who forces a win, first mover or second?

One more Conjecture

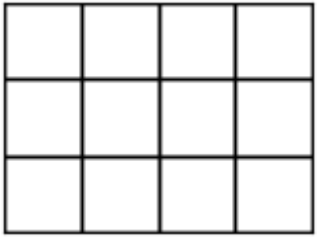

Figure: Who forces a win, first mover or second?

Conjecture

*For empty boards of size m*n, the (allsmall) game is 0 if m*n is even, else the game is* $*$.

Thank You

Thank you, any questions are welcome

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